UNIVERSITY OF EDUCATION
"UExam" Semester-II, 2019
M.Sc Mathematics Session:2018-20

No. 101

Roll No. (in fig.)__________

Roll No. (in words)__________

Candidate's Signature.

Signature of Addl. Supdt.

Course Code: MATH3119
Subject: Introduction to Topology

SECTION: I (MCQ's)

Time Allowed: 20 Minutes **Max. Marks: 18**

NOTE: Encircle the correct/ best answer in each of the followings. Each Question carries 1 mark. Use of remover carries zero mark. Cutting and Overwriting is not allowed.

Q1.

- Let X = [0, 1]. The interval (0, 1/2) is ------------ in X.
  a) Open b) closed c) semi open d) none
- The union of ----------- collection of open sets is open.
  a) Open b) any c) finite d) none
- The ------------------ forms an equivalence relation on the class of all topological spaces.
  a) Homeomorphism b) Isomorphism c) bijective d) injective
- A subset A of a topological space X is closed iff A contains each of its --------- point.
  a) Limit b) exterior c) interior d) closed
- Let A and B be the subsets of topological space (X, τ), then $(AUB)' = -----$
  a) $A'UB$ b) $A'UB'$ c) A' d) B'
- $\bar{Q}$ = -----------
  a) Q b) Z c) R d) none
- $A^\circ UB^\circ$ ------- $(AUB)^\circ$
  a) $\subset$ b) $\supset$ c) $\supseteq$ d) =
- A is open iff A ---------A°
  a) $\subset$ b) $\supset$ c) $\supseteq$ d) =
- The ----------- intervals form a base for the usual topology on R.
  a) Open b) any c) finite d) none
- Topology in another way is the ------------geometry.
  a) Qualitative b) quantative c) analytical d) none
- The term used to describe two geometric objects that are topologically equivalent is-------
  a) Homeomorphic b) Isomorphic c) bijective d) all
- Co-finite topology of a finite set is a ------------topology.
  a) discrete b) Indiscrete c) finite d) all
- If τ_1 and $\tau_2$are two topologies on non-empty set X, then .... is topological space.
  a) $\tau_1 U \tau_2$ b) $\tau_1 \cap \tau_2$ c) $\tau_1 \backslash \tau_2$ d) $\tau_2 \backslash \tau_1$
- If τ is typology on non-empty set X, then arbitrary .... of member of τ belong to τ
  a) union b) intersection c) product d) compliment
- A subset A of a topological space X is said to be ---------------in X iff $\bar{A}$= X.
  a) nowhere dense b) dense c) countable d) none
- The interior of the compliment of A is called the -------------of A.
  a) open set b) closed set c) Exterior d) none
- If $\tau_1 \subset \tau_2$ Then we say is τ_1 --------------- than τ_2.
  a) Coarser b) weaker c) smaller d) all
- The class of all singletons of a set forms a --------------for the discrete topology.
  a) base b) basis c) both d) none

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Max. Marks: 42

Time Allowed: 100 Minutes.

Section II (Short Answer)

Q.2- Write short answers of the following. **3x6 = 18**

i. Find all possible topologies on X = {a, b, c}.
ii. Consider the following set consisting of three points; X = {a, b, c} and determine does the set $\tau = \{\emptyset, X, \{a\}, \{b\}\}$ satisfy the requirements for a topology?
iii. Define Discrete topology.
iv. Let A = {1, 1/2, 1/3, 1/4, ...} is a subset of R. Prove that A is nowhere dense in R.
v. Co-finite topology of a finite set is a discrete topology.
vi. Let X = {a, b, c, d, e}, and $\tau = \{\varphi, X, \{a\}, \{c, d\}, \{a, c, d\}, \{b, c, d, e\}\}$ be a topology on X. Consider the set A = {a, b, c}.Find its derived set?

Section III (Essay Type)

Answer the following Questions **6x4 = 24**

Q.3: - A function f: X → Y is continuous iff the inverse image of every closed subset of Y is closed in X.

Q.4: - Let A be a subset of a topological space X. The closure of A is the intersection of all closed supersets of A. i.e. Closure of A is the smallest closed superset of A. Closure of a set A is denoted by $\bar{A}$.

Q.5: - Let X be a topological space. Then the class of closed subsets of X possesses the following properties.

1. φ and X are closed sets.
2. The intersection of any number of closed sets is closed.
3. The union of any two closed sets is closed.

Q.6: - Let $(X, \mathcal{T})$ be a topological space and let A, B be any subsets of X then $(A \cap B)^\circ = A^\circ \cap B^\circ$.

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"UExam" Semester-II, 2019
Msc. Mathematics Session:2018-20

Course Code: MATH3117
Subject: Real Analysis-II

Time Allowed: 100 Minutes.

Supdt.

Max. Marks: 42

Section II (Short Answer)

Q.2- Write short answers of the following. **3x6 = 18**

i. Give an example of sequence of functions, which converges point wise but does not converge uniformly.

ii. Let

$$f_n(x) = \begin{cases} n^2x, & \text{for } 0 \le x \le \frac{1}{n} \\ -n^2(x-2/n), & \text{for } \frac{1}{n} \le x \le \frac{2}{n} \\ 0, & \text{for } \frac{1}{n} \le x \le 1 \end{cases}$$

Justify that $\lim\limits_{n\to\infty}\int_0^1 f_n \ne \int_0^1 \lim\limits_{n\to\infty} f_n$.

iii. Prove that the Riemann's Integral of a function if exists is uniquely determined.

iv. Discuss point-wise and uniform convergence of $f_n : (-\infty,+\infty) \to (-\infty,+\infty)$ defined as $f_n(x) = x^n$.

v. Prove that the sum of two Riemann integrable functions is Riemann integrable.

vi. Prove that every Riemann's Integrable function is bounded. Give an example to show that converse of this statement does not hold.

Section III (Essay Type)

Answer the following Questions **4x6 = 24**

Q.no.3 Define Daurbox integral and prove that a function is Daurbox integrable if and only if it is Riemann integrable.

Q.no.4 Prove that every continuous function defined on closed interval is Riemann integrable. What about its converse.

Q.no.5 Let (f_n) be a sequence of bounded functions on $A \subseteq R$. Then prove that this sequence converges uniformly to a function f if and only if for each $\varepsilon > 0$ there is a number $H(\varepsilon)$ in N such that for all $m,n \ge H(\varepsilon)$, $\|f_m - f_n\| < \varepsilon$.

Q.no.6 Let $g:[0,3] \to (-\infty,+\infty)$ be a function defined as

$$g(x) = \begin{cases} 3 & \text{for } 0 \le x \le 1, \\ 4 & \text{for } 1 < x \le 3. \end{cases}$$

Find its Riemann integral by using definition.

UNIVERSITY OF EDUCATION
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M.Sc Mathematics Session:2018-20

No. 108

Course Code: MATH3118
Subject: Number Theory

Roll No. (in fig.)________

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SECTION: I (MCQ's)

Time Allowed: 20 Minutes Max. Marks: 18

NOTE: Encircle the correct/ best answer in each of the followings. Each Question carries 1 mark. Use of remover carries zero mark. Cutting and Overwriting is not allowed.

Q1. 8

- The gcd of two numbers 272 and 1479 is
  a) 17 b) 13 c) 19 d) 9
- The linear diaphontine equation $172x+20y=1000$ has a solution if and only if
  a) $2|1000$ b) $4|1000$ c) $8|1000$ d) All of these
- The prime factorization of the integer 10140 is
  a) $2^2.3^2.13^2.5$ b) $2^2.3.13^2.5^2$ c) $2^2.3^2.13.5$ d) $2^2.3.13^2.5$
- 1949 and 1951 are
  a) Twin primes b) Mersenn primes c) Fermat prime d) Both (a) and (b)
- When 2^{50}. is divided by 7, it gives remainder
  a) 5 b) 4 c) 2 d) 1
- The integer 1010908899 is divisible by....
  a) 7 b) 11 c) 13 d) 3
- The no of the form 2^n-1, $n\geq 1$ is......
  a) Pseudo prime b) Fermat prime c) Mersenn prime d) None of these
- 19/51 has continued fraction of the form......
  a) [0; 2, 1,2, 6] b) [0; 2, 1,2, 5, 1] c) [0; 2, 2,1, 6] d) Both (a) and (b)
- An odd prime p is expressible as a sum of two squares iff .....
  a) $p\equiv 3 \pmod 4$ b) $p\equiv 1 \pmod 4$ c) $p\equiv 2 \pmod 4$ d) All of these
- The number 65537 is called.....
  a) Fermat prime b) Mersenn Prime c) Pseudo prime d) None
- For p=19 there are .... pairs of quadratic residues
  a) 4 b) 2 c) 3 d) 5
- The number 13 has exactly .....primitive roots
  a) $\varphi(12)$ b) $\varphi(11)$ c) $\varphi(10)$ d) No
- $\left\lceil \frac{\pi-\lfloor e\rfloor+3}{\lfloor e\rfloor} \right\rceil$=......
  a) 3 b) 2 c) -2 d) -3
- $[72, 44]$ =......
  a) 788 b) 798 c) 792 d) 784
- The number 1009 is
  a) An odd prime b) Fermat prime c) Mersenn prime d) None of these
- $\pi(15)=$......
  a) 6 b) 5 c) 4 d) 3
- $Z_p=\{0,1,2,....p-1\}$ forms a
  a) (G, +) b) Field c) Ring d) (G, .)
- $\tau(\phi(31))=$ .....
  a) 7 b) 15 c) 8 d) 4

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Section II (Short Answer)

Q.2- Write short answers of the following. 3x6 = 18

i. Verify that 1951 and 1949 are twin primes.
ii. Enlist all the primitive roots of 19.
iii. Find the last two digits of 7^{100} in its decimal expansion.
iv. Show that the congruence relation is an equivalence relation.
v. Show that 1387 is a Pseudo Prime.
vi. Obtain all the Quadratic residues mod 17.

Section III (Essay Type)

Answer the following Questions 6x4 = 24

Q.3:- State and prove "LIFTING LEMMA"

Q.4:- The linear congruence $ax \equiv b \pmod{m}$ has a solution if and only if $d \mid b$. where $d = \gcd(a,b)$. If $d \mid b$, then it has d mutually incongruent solutions modulo m.

Q.5:- Find all possible solution(s) of $x^2 + x + 3 \equiv 0 \ (mod\ 3^3)$

Q.6:- If p is a prime then the congruence $f(x) \equiv 0 \pmod{p}$ of degree n has at most n solutions.