

UNIVERSITY OF EDUCATION

"UExam" Semester-II, 2019

M.Sc Mathematics Session:2018-20

Course Code: MATH3118

Subject: Number Theory

SECTION: I (MCQ's)

Time Allowed: 20 Minutes

Max. Marks: 18

NOTE: Encircle the correct/ best answer in each of the followings. Each Question carries 1 mark. Use of remover carries zero mark. Cutting and Overwriting is not allowed.

No. 108

Roll No. (in fig.) \_\_\_\_\_

Roll No. (in words) \_\_\_\_\_

Candidate's Signature. \_\_\_\_\_

Signature of Addl. Supdt. \_\_\_\_\_

Q1.

- The gcd of two numbers 272 and 1479 is
  - a) 17
  - b) 13
  - c) 19
  - d) 9
- The linear diophantine equation  $172x+20y=1000$  has a solution if and only if
  - a)  $2|1000$
  - b)  $4|1000$
  - c)  $8|1000$
  - d) All of these
- The prime factorization of the integer 10140 is
  - a)  $2^2 \cdot 3^2 \cdot 13^2 \cdot 5$
  - b)  $2^2 \cdot 3 \cdot 13^2 \cdot 5^2$
  - c)  $2^2 \cdot 3^2 \cdot 13 \cdot 5$
  - d)  $2^2 \cdot 3 \cdot 13^2 \cdot 5$
- 1949 and 1951 are
  - a) Twin primes
  - b) Mersenn primes
  - c) Fermat prime
  - d) Both (a) and (b)
- When  $2^{50}$  is divided by 7, it gives remainder
  - a) 5
  - b) 4
  - c) 2
  - d) 1
- The integer 1010908899 is divisible by....
  - a) 7
  - b) 11
  - c) 13
  - d) 3
- The no of the form  $2^n - 1, n \geq 1$  is.....
  - a) Pseudo prime
  - b) Fermat prime
  - c) Mersenn prime
  - d) None of these
- $19/51$  has continued fraction of the form.....
  - a)  $[0; 2, 1, 2, 6]$
  - b)  $[0; 2, 1, 2, 5, 1]$
  - c)  $[0; 2, 2, 1, 6]$
  - d) Both (a) and (b)
- An odd prime  $p$  is expressible as a sum of two squares iff ....
  - a)  $p \equiv 3 \pmod{4}$
  - b)  $p \equiv 1 \pmod{4}$
  - c)  $p \equiv 2 \pmod{4}$
  - d) All of these
- The number 65537 is called....
  - a) Fermat prime
  - b) Mersenn Prime
  - c) Pseudo prime
  - d) None
- For  $p=19$  there are ..... pairs of quadratic residues
  - a) 4
  - b) 2
  - c) 3
  - d) 5
- The number 13 has exactly .....primitive roots
  - a)  $\phi(12)$
  - b)  $\phi(11)$
  - c)  $\phi(10)$
  - d) No
- $\left\lfloor \frac{\pi - \lfloor e \rfloor + 3}{\lfloor e \rfloor} \right\rfloor = \dots\dots$ 
  - a) 3
  - b) 2
  - c) -2
  - d) -3
- $[72, 44] = \dots\dots$ 
  - a) 788
  - b) 798
  - c) 792
  - d) 784
- The number 1009 is
  - a) An odd prime
  - b) Fermat prime
  - c) Mersenn prime
  - d) None of these
- $\pi(15) = \dots\dots$ 
  - a) 6
  - b) 5
  - c) 4
  - d) 3
- $Z_p = \{0, 1, 2, \dots, p-1\}$  forms a
  - a)  $(G, +)$
  - b) Field
  - c) Ring
  - d)  $(G, \cdot)$
- $\tau(\phi(31)) = \dots\dots$ 
  - a) 7
  - b) 15
  - c) 8
  - d) 4

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**"UEXAM" Semester-II, 2019**  
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Time Allowed: 100 Minutes.

Max. Marks: 42

**Section II (Short Answer)**

**Q.2-** Write short answers of the following.

**3x6 = 18**

- i. Verify that 1951 and 1949 are twin primes.
- ii. Enlist all the primitive roots of 19.
- iii. Find the last two digits of  $7^{100}$  in its decimal expansion.
- iv. Show that the congruence relation is an equivalence relation.
- v. Show that 1387 is a Pseudo Prime.
- vi. Obtain all the Quadratic residues mod 17.

**Section III (Essay Type)**

**Answer the following Questions**

**6x4 = 24**

**Q.3:-** State and prove "LIFTING LEMMA"

**Q.4:-** The linear congruence  $ax \equiv b \pmod{m}$  has a solution if and only if  $d \mid b$ .  
where  $d = \gcd(a, m)$ . If  $d \mid b$ , then it has  $d$  mutually incongruent solutions modulo  $m$ .

**Q.5:-** Find all possible solution(s) of  $x^2 + x + 3 \equiv 0 \pmod{3^3}$

**Q.6:-** If  $p$  is a prime then the congruence  $f(x) \equiv 0 \pmod{p}$  of degree  $n$  has at most  $n$  solutions.